AA-2

A Study of Sub-Millimeter Wave Coupled Dielectric Waveguides Using the GIE Method

T.E. van Deventer, P.B. Katehi

Radiation Laboratory, EECS Department, University of Michigan, Ann Arbor, MI

Abstract-

In this paper the coupling properties of coupled dielectric waveguides are evaluated using a novel and powerful method which relies on the concept of equivalent planar polarization dipole moments to simulate the guides. Generalized impedance boundary conditions are enforced to provide a simple planar integral equation (Generalized Integral Equation). This method can account for multiple dielectric strips on different levels. Phase constants of the different modes and coupling characteristics are calculated for several structures, such as rib waveguides and insulated image guides.

1 INTRODUCTION

New waveguiding low-loss monolithic transmission lines have been proposed recently for sub-millimeter wave applications [1]. They exhibit several advantages over more conventional conducting lines such as: low ohmic losses, electrically small size (fraction of a guided wavelength), good guiding properties by appropriate combination of layers, easy fabrication, and monolithic nature that allows for easy construction of passive circuit elements as well as simple integration of active devices. The theoretical characterization of this new type of dielectric structures plays an important role in the design of low-loss circuits operating in the sub-millimeter wave region, such as power dividers, impedance transformers and filters.

The fabrication of geometrically complex circuits on multilayered substrates faces stringent requirements on the spacing between elements. This in turn requires a good understanding of various coupling mechanisms. During the past few years, a number of papers have been published on the characterization of edge-coupled dielectric lines at millimeter-wave frequencies using the effective dielectric constant (EDC) method [2], [3], the mode-matching technique [4], variational methods [5] and integral equation formulations [6], [7].

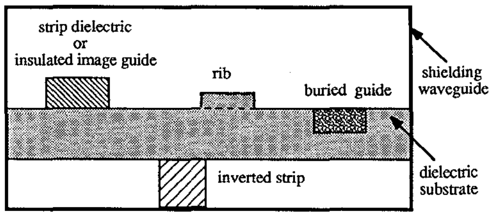


Figure 1: General configuration of multiple low-loss ridged waveguides

The present paper shows an important extension of the novel generalized integral equation (GIE) method described in [8] to characterize complex geometries at submillimeter-wave and terahertz frequencies. Multiple dielectric strips on different levels within a multilayered substrate environment can be analyzed with accuracy and simplicity. With the replacement of the dielectric strips by planar equivalent currents, the original problem is simplified and can be treated as any other two-dimensional problem with unknown planar current densities [9]. An accurate modelling of the actual coupling between multilevel lines, including the effect of dispersion at high frequencies, is presented.

2 THEORY

We consider the dielectric structure within a shielded metallic waveguide (Figure 1). The presence of the waveguide does not affect the guiding properties of the lines when the walls are far enough. Two types of modes may then propagate in the dielectric structure, namely waveguide modes (known as surface wave modes in open configuration) which are related to the supporting structure, and strip modes which are confined to the dielectric waveguides. The first modes will always propagate above the cut-off of the waveguide. On the other hand, the strip mode will not exist unless the guiding layer is above a critical thickness and width. In this analysis, the width-to-thickness ratio of the strips is moderately large. Under this assumption, the eigenvalue equation for the propagation constant k_z is derived through the use of a modified planar integral equation employing the

AA

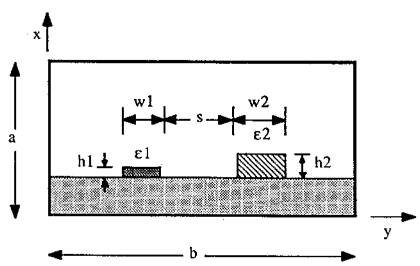
concept of generalized boundary conditions. Generalized impedance boundary conditions have been used earlier for the solution of scattering problems and have provided novel formulations [10]. Throughout this paper, both conductors (ground plane and shielding waveguide) and dielectrics (substrates and strips) are assumed lossless, but the effect of losses can be accurately modeled [9].

The approach consists of a generalized integral formulation where the electric field is derived in terms of equivalent electric polarization currents. Consider a multilevel structure comprising several dielectric strips ($i = 1, 2, ..., N$) of permittivity ϵ_i with nonmagnetic material and with thickness h_i a fraction of the wavelength in the dielectric as shown in Figure 2a. The excitation of an electromagnetic field gives rise to an electric dipole moment per unit volume in strip i denoted by $\vec{P}^i$ which is also known as the *polarization vector*. This electric dipole moment per unit volume is given by:

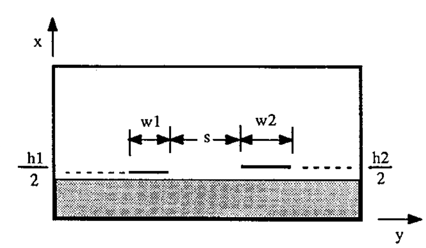
$$\vec{P}^i = \begin{cases} (\epsilon_i - \epsilon_o)\,\vec{E}_i & , \ in\ S_i \\ 0 & , \ elsewhere. \end{cases} \qquad (1)$$

First, we define an equivalent problem with respect to the field outside the dielectric strips. Each strip is replaced by a planar-strip polarization surface resting on the plane $x_i = \frac{h_i}{2}$ (Figure 2b) and is characterized by a *dipole moment per unit surface* $\vec{P}_s^i$ given by:

$$\vec{P}_s^i(x_i, y) = \int_{-h_i}^{x_i} \vec{P}^i(x', y)\, dx' + \int_{x_i}^{h_i} \vec{P}^i(x', y)\, dx' , \qquad (2)$$



a. General geometry



b. Equivalent geometry with dipole moments

Figure 2: Equivalent problem of the analyzed structure

or

$$\vec{P}_s^i(x_i, y) = \vec{P}_s^{i+}(x_i, y) + \vec{P}_s^{i-}(x_i, y) . \qquad (3)$$

From equations (1) and (2) and by using Taylor's expansion for the fields inside the dielectric strips we can express the dipole moments per unit surface $\vec{P}_s^i$ in terms of the higher order derivatives of this electric field as shown below:

$$\vec{P}_s^i = (\epsilon_i - \epsilon_o) \sum_{n=0}^{\infty} \frac{h_i^{n+1}}{\Gamma(n+2)} \left\{ (-1)^n \frac{\partial^n}{\partial x^n}\bigg|_{x=h_i} + \frac{\partial^n}{\partial x^n}\bigg|_{x=-h_i} \right\} \vec{E}_{ds}^i(x, y) \qquad (4)$$

where $\vec{E}_{ds}^i$ is the field inside the ith dielectric strip.

In the presence of the planar polarization surface, the radiated electric field $\vec{E}_p$ is given by the following integral:

$$\vec{E}_p(x, y) = \sum_{i=1}^{N} \int_{L_p} \left[\bar{\bar{G}}_p^{ki}(x, y/x_p, y') \right] \cdot \vec{P}_s^i(x_p, y')\, dy' \qquad (5)$$

where $\bar{\bar{G}}_p^{ki}$ is the dyadic green's function on line k due to line i and N is the total number of strips. As it is well known, this function can be found analytically and will be in the form of infinite single summations for shielded structures and single Sommerfeld integrals for open structures.

In order to make the above two problems equivalent in the volume outside the dielectric strips, the total field $\vec{E}_p$ has to be identical to the original $\vec{E}^i$ field on the surface of the dielectric waveguides. Furthermore, using the generalized boundary conditions on the upper and lower surfaces of the dielectric strips, we find that the normal derivatives of the outside electric field $\vec{E}^i$ on these surfaces are related to the normal derivatives of the electric field $\vec{E}_{ds}^i$ excited inside the dielectric strip by

$$\frac{\partial^n \vec{E}^i}{\partial x^n} = \bar{\bar{\mathcal{C}}} \cdot \left\{ \frac{\partial^n \vec{E}_{ds}^i}{\partial x^n} + \vec{\mathcal{R}}(\vec{E}_{ds}^i) \; k_0^2(\epsilon_o - \epsilon_i) \sum_{m=1}^{N} (-F)^{m-1} \frac{\partial^{n-2m} \vec{E}_{ds}^i}{\partial x^{n-2m}} \right\} \qquad (6)$$

where $\bar{\bar{\mathcal{C}}}$ is a known tensor, $\vec{\mathcal{R}}$ is a vector function involving first order x-derivatives of the x-component of the field and

$$F = \frac{\partial^2}{\partial y^2} + k_d^2 - k_z^2 . \qquad (7)$$

In view of the above equations, the equivalent strip dipole moments per unit surface can be expressed in terms of the outside electric fields at the upper and lower strip surfaces as

$$\vec{P}_s = \mathcal{L}\left\{\vec{E}, \frac{\partial \vec{E}}{\partial x}, \frac{\partial^2 \vec{E}}{\partial x^2}, \cdots\right\}_{x=\pm h_i} , \quad (8)$$

which also indicates that the equivalent polarization surface exhibits spatially dispersive characteristics. When the operator $\mathcal{L}$ of equation (8) is applied to the original $\vec{E}^i$ field, it transforms equation (5) to a homogeneous *Fredholm Integral Equation of the second kind*:

$$\vec{P}_s^k = \sum_{i=1}^{N} \int_{L_p} \bar{\bar{G}}_{pmod}^{ik} \cdot \vec{P}_s^i \, dy' , \quad (9)$$

which may be solved to determine the unknown equivalent planar currents. Because the structure is in a shielded environment, radiation losses are not present. However, the procedure as described up to this point can be applied as well to **open** dielectric waveguide problems with the presumption that the boundary conditions away from the dielectric strip surfaces are satisfied appropriately by the Green's functions $\bar{\bar{G}}_p^{ki}$. In both cases the application of the generalized boundary conditions result in infinite summations which can be evaluated analytically leading to simplified kernels.

The method of moments is applied to the dipole moments $\vec{P}_s^i$ to solve (9). Subsectional pulse basis functions are chosen as expansion functions for the transverse dependence of the dipole moments. Galerkin's method is applied to transform the integral equation into a matrix equation whose eigenvalues are the propagation constant of the different modes at the operating frequency.

The present study also involves the calculation of the coupling coefficient. In the case of two symmetric coupled lines, the coupling coefficient is defined according to classical coupled mode theory [11]. For maximum coupling, the coupler should be designed with a length of

$$L = \frac{\pi}{\beta_e - \beta_o} \quad (10)$$

where β_e and β_o are the phase constants of the even and odd mode, respectively.

3 RESULTS AND DISCUSSION

A computer program was implemented to calculate the propagation constant and coupling coefficient of multilevel lines using the approach described above. Figure 3 shows the dispersion characteristics of coupled dielectric lines in a horizontal (E_y) field configuration. The dotted lines corespond to the first two waveguide modes of the partially-filled structure (LSE_{10} and LSE_{20}, in this example) and the solid lines to the modes of the structure with the strips present. For high frequencies, the odd mode is actually higher than the even mode because of the waveguide polarization in this particular example. As the operating frequency decreases, the strips become electrically small and the fields are no more confined to the strips. The corresponding modes then degenerate to a perturbation of the partially-filled waveguide modes. A number of non-physical modes without low-frequency cut-off were found. The pattern of these spurious modes was easily recognizable and was disgarded on Figure 3 for sake of clarity (as were the many higher-order modes propagating above 250 GHz). This type of problem is not uncommon in the numerical solution of electromagnetics problems, as in the case of the finite-element method [12].

In Figure 4, the phase constant of the odd and even modes are investigated for three different types of guides : the rib ($\epsilon_{guide} = \epsilon_{substrate}$), the strip dielectric ($\epsilon_{guide} < \epsilon_{substrate}$) in region 1 and the insulated image guide ($\epsilon_{guide} > \epsilon_{substrate}$) in region 2. Note that for $\epsilon_{guide} = 1$, no strip is actually present, and the phase constant reduces to the waveguide mode of the partially-filled structure. The permittivity of the substrate was chosen to be 2.2 in this example. However, in actual fabrication, III-V materials must be selected for the substrate and lines due to their adhesion properties. The phase constant is shown (Figure 5) for two identical strip dielectric guides as a function of separation s and compared to a single line at the location of strip #1. The normalized propagation constant of the odd and even modes tends to degenerate to the single line case as the separation increases, showing a decrease in the coupling between the lines. This is due to the fact that at higher frequencies the fields tend to concentrate in the strip regions. The normalized coupling length L/h is plotted in Figure 6, where h is the height of the dielectric guide. The higher the normalized wavenumber of the waveguide, the higher is the coupling length.

This work will be extended to study the effect of losses on the performance of thin dielectric lines at high frequencies. In addition, results for multilevel lines of different thicknesses will be presented as a function of line separation and frequency.

4 CONCLUSIONS

Several types of shielded dielectric waveguides are analyzed using the generalized integral equation method (GIE). This method employs a dyadic Green's function and integral equation formulation together with higher-order boundary conditions to study the influence of frequency, material constants and geometrical dimensions on the propagation constants of coupled dielectric strip waveguides.

5 ACKNOWLEDGMENTS

This work was supported by NASA/Center for Space Terahertz Technology at the University of Michigan and by the Army Research Office under contract project DAAL03-91-G-0116.

References

[1] A.G. Engel Jr., P.B. Katehi, "Low-loss Monolithic Transmission Lines for Sub-Mm and Terahertz Frequency Applications," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-39, No. 11, pp. 1847-1854, November 1991.

[2] T. Itoh, "Inverted Strip Dielectric Waveguide for Millimeter-Wave Integrated Circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, No. 11, pp. 821-827, November 1976.

[3] T. Trinh, R. Mittra, "Coupling Characteristics of Planar Dielectric Waveguides of Rectangular Cross Section," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, No. 9, pp. 875-880, September 1981.

[4] U. Crombach, "Analysis of Single and Coupled Rectangular Dielectric Waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, No. 9, pp. 870-874, September 1981.

[5] K. Hayata, M. Koshiba, M. Eguchi, M. Suzuki "Vectorial Finite-Element Method without any Spurious Solutions for Dielectric Waveguiding Problems using Transverse Magnetic-field Component" *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, No. 10, pp. 1120-1124, November 1986.

[6] J.S. Bagby, D.P. Nyquist, and B.C. Drachman, "Integral Formulation for Analysis of Integrated Dielectric Waveguides" *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, No. 10, pp. 906-915, October 1985.

[7] J.F. Kiang, S.M. Ali, J.A. Kong, "Integral Equation Solution to the Guidance and Leakage Properties of Coupled Dielectric Strip Waveguides" *IEEE Trans. Microwave Theory Tech.*, vol. MTT-38, No. 2, pp. 193-203, February 1990.

[8] T.E. van Deventer, P.B. Katehi, "A Planar Integral Equation for the analysis of Dielectric Ridge Structures Using Generalized Boundary Conditions", *IEEE MTT-International Symposium Digest*, Boston, pp. 647-650, June 1991.

[9] T.E. van Deventer, P.B. Katehi, A.C. Cangellaris, "An Integral Equation Method for the Evaluation of Conductor and Dielectric Losses in High Frequency Interconnects," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-37, No. 12, pp. 1964-1972, December 1989.

[10] T.B.A. Senior, J.L. Volakis, "Generalized Impedance Boundary Conditions in Scattering," *Proceedings of the IEEE*, vol. 79, No. 10, pp. 1413-1420, October 1991.

[11] D. Marcuse, "Theory of Dielectric Optical Waveguides", 2nd edition, Academic Press, Inc., 1991.

[12] B.M.A. Rahman, J.B. Davies, "Penalty function Improvement of Waveguide Solution by Finite Elements", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, No. 8, pp. 922-928, August 1984.

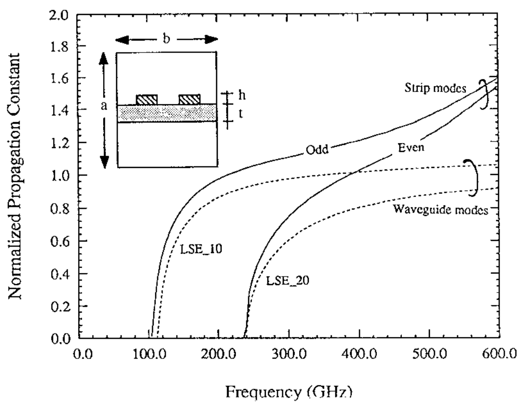


Figure 3: Normalized phase constant of the propagating modes as a function of frequency (a = 1.25mm, b = 500μm, $w_1 = w_2$ = 125μm, $h = h_1 = h_2$ = 50μm, t = 50μm, s = 125μm, ϵ_l = 12, ϵ_{sub} = 2.2)

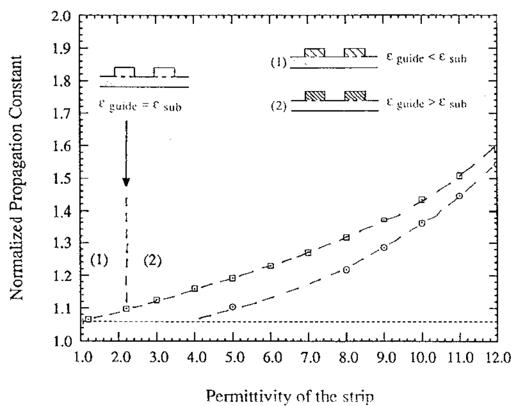


Figure 4: Propagation constant as a function of permittivity of the line (a = 5cm, b = 2cm, $w_1 = w_2$ = 5mm, $h_1 = h_2$ = 2mm, s = 5mm, ϵ_{sub} = 2.2, f = 15 GHz)

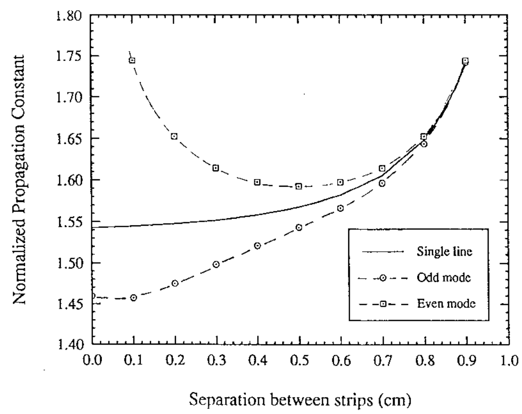


Figure 5: The normalized phase constant as a function of strip separation (a = 5cm, b = 2cm, $w_1 = w_2$ = 5mm, $h_1 = h_2$ = 2mm, ϵ_l = 12, ϵ_{sub} = 2.2, f = 15 GHz)

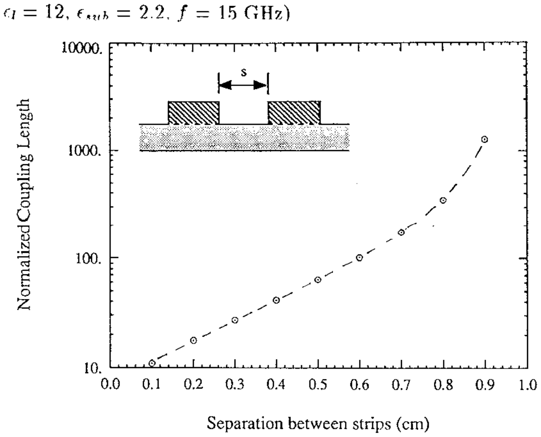


Figure 6: The normalized coupling length for total power transfer versus strip separation